Gen3 Lite Robot library manual v1.0

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1 Gen3 Lite Robot library

The *Gen3 Lite Robot* library is devised to facilitate the implementation of control algorithms in Matlab® and Simulink® using the dynamic model of the 6-DoF Gen3 Lite robot developed by Kinova Inc. The dynamic model has been derived with the Euler-Lagrange equations of motion [1] and the physical parameters of the robot described in [3]. The library uses C++ MEX functions for Windows® OS (operating system) [2]. Figure 1 shows the blocks that compose the library, and the mask of the principal block is detailed in Figure 2.

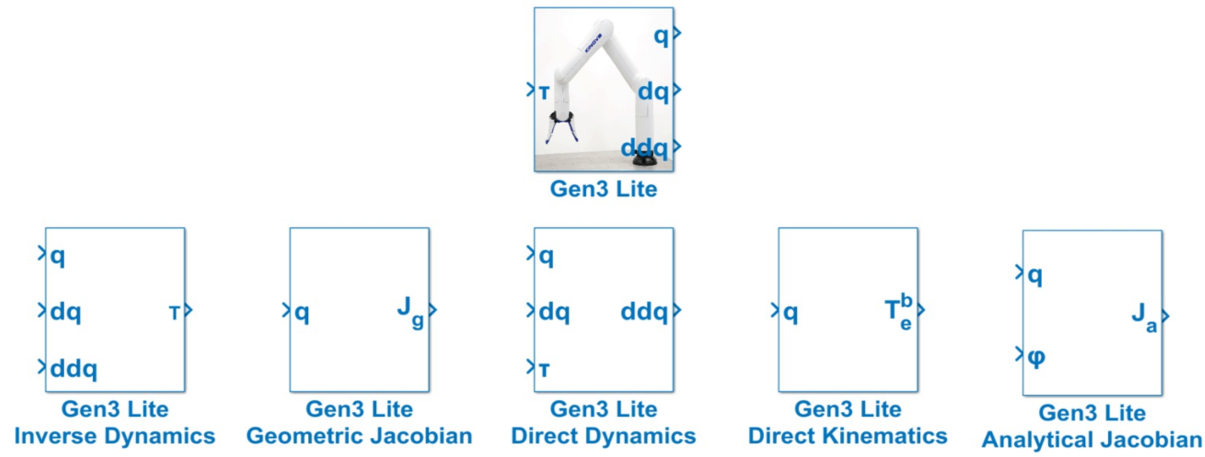

Figure 1: *Gen3 Lite* Library v1.0

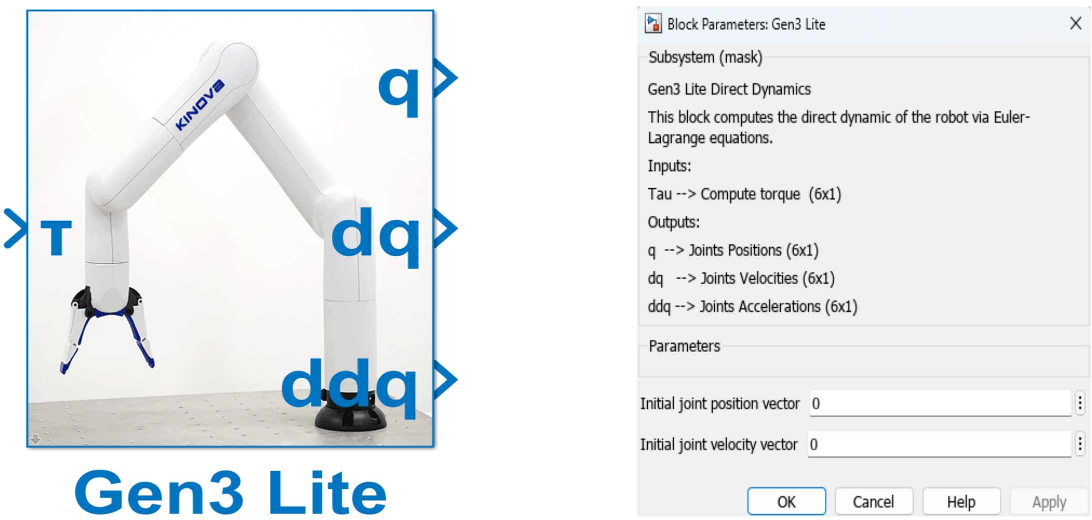

Figure 2: Mask of the Gen3 Lite block, the parameters initial joint positions and initial joint velocities, receives $\mathbf{q}(0)$, and $\dot{\mathbf{q}}(0)$, respectively.

How to use the Library

Using the library is easy, first in the MATLAB workspace unzip the Gen3-Lite-Robot-Library.zip. Open the Gen3 Lite Library.sxl, and in the Simulink project copy and paste the block needed for your simulation (make sure that the project is in the same folder as the library). The Gen3_Lite_example.sxl file shows a simple example of a PD controller (see Figure 3).

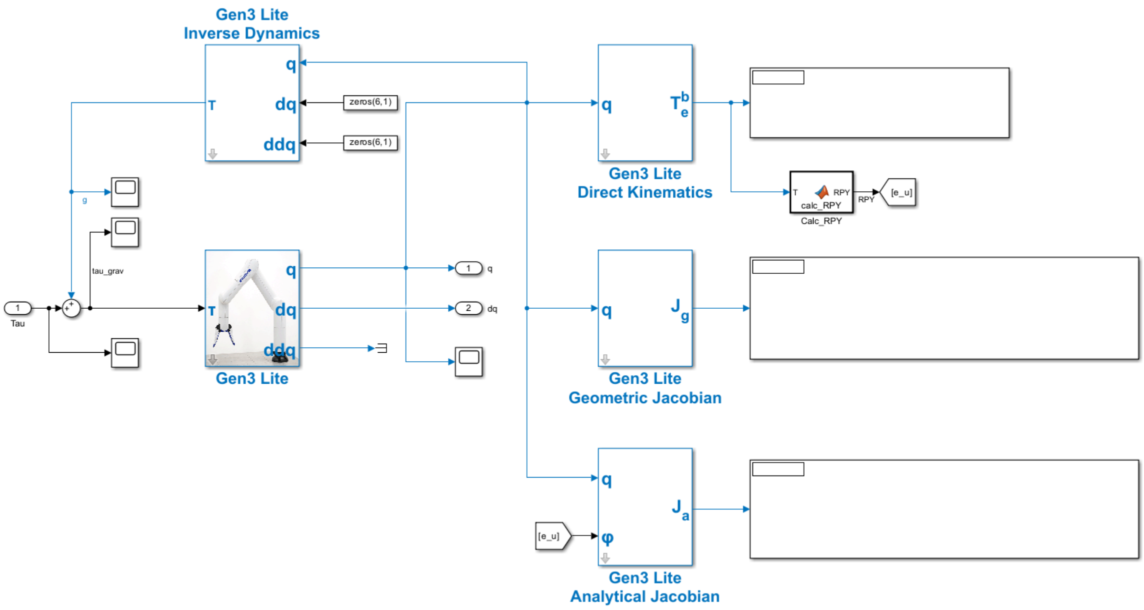

Figure 3: Example of a PD controller.

As mentioned above, the library is for Windows OS, in case the user is on Linux or macOS, the folder cpp_files contains the C++ files in order that the user can compile the functions for the specific operating system. Figure 4 shows the simple command to create a MEX function, and table 1 shows the MEX File Platform-Dependent Extension[1].

```
>> mex calc_direct_kinematics.cpp
Building with 'MinGW64 Compiler (C++)'.
MEX completed successfully.
```

Figure 4: Example of how to compile a MEX function.

Platform	Binary MEX File Extension
Windows	mexw64
Linux	mexa64
macOS with Apple silicon	mexmaca64
macOS with Intel®	mexmaci64

Table 1: MEX File Platform-Dependent Extension.

[1]See the tutorial in [2]

Kinematics and Jacobian

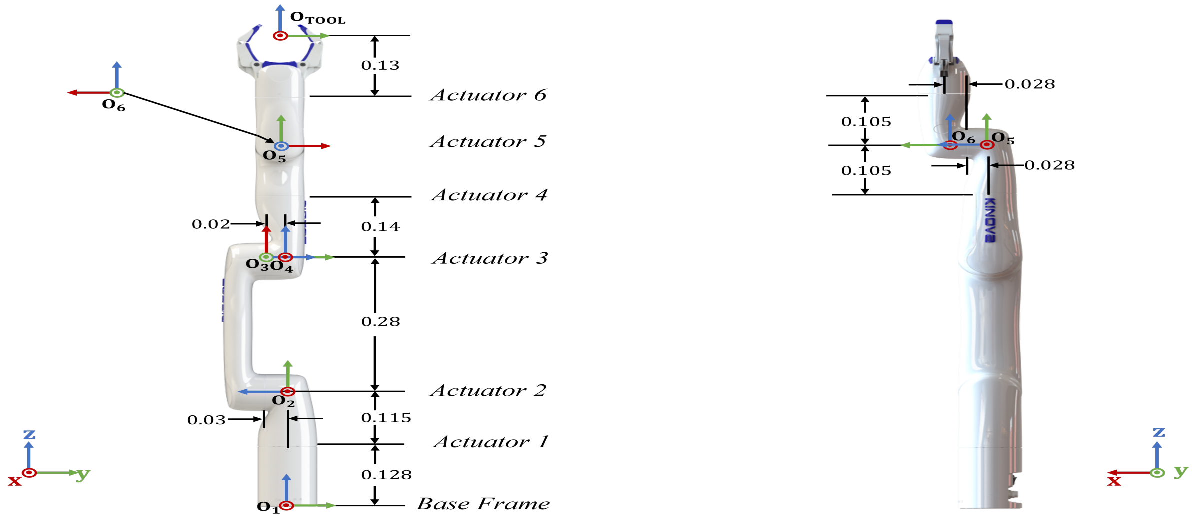

Figure 5: Gen3 Lite robot.

Link	$\alpha_i\,[rad]$	$a_i\,[m]$	$d_i\,[m]$	$q_i\,[rad]$
1	$\pi/2$	0	$(0.128 + 0.115)$	q_1
2	π	0.28	0.03	$q_2 + \pi/2$
3	$\pi/2$	0	0.02	$q_3 + \pi/2$
4	$\pi/2$	0	$(0.140 + 0.105)$	$q_4 + \pi/2$
5	$\pi/2$	0	$(0.285 + 0.285)$	$q_5 + \pi$
6	0	0	$(0.105 + 0.130)$	$q_6 + \pi/2$

Table 2: DH-Parameters of Gen3 Lite robot.

The direct kinematics of the 6-DoF Gen3 Lite robot shown in Figure 5 is obtained with the DH-Parameters of the Table (2), therefore the direct kinetics is given by

$$\mathbf{T}_{O_{TOOL}}^{O_1}(\mathbf{q}) := \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{1}$$

where

$$\begin{aligned}
r_{11} =& -s_6(c_5(c_4s_1 + c_{23}c_1s_4) + s_{23}c_1s_5) - c_6(s_1s_4 - c_{23}c_1c_4);\\
r_{12} =& s_6(s_1s_4 - c_{23}c_1c_4) - c_6(c_5(c_4s_1 + c_{23}c_1s_4) + s_{23}c_1s_5);\\
r_{13} =& s_5(c_4s_1 + c_{23}c_1s_4) - s_{23}c_1c_5;\\
r_{21} =& s_6(c_5(c_1c_4 - c_{23}s_1s_4) - s_{23}s_1s_5) + c_6(c_1s_4 + c_{23}c_4s_1);\\
r_{22} =& c_6(c_5(c_1c_4 - c_{23}s_1s_4) - s_{23}s_1s_5) - s_6(c_1s_4 + c_{23}c_4s_1);\\
r_{23} =& -s_5(c_1c_4 - c_{23}s_1s_4) - s_{23}c_5s_1;\\
r_{31} =& s_6(c_{23}s_5 - s_{23}c_5s_4) + s_{23}c_4c_6;\\
r_{32} =& c_6(c_{23}s_5 - s_{23}c_5s_4) - s_{23}c_4s_6;\\
r_{33} =& c_{23}c_5 + s_{23}s_4s_5;\\
p_x =& d_6(s_5(c_4s_1 + c_{23}c_1s_4) - s_{23}c_1c_5) - d_5(s_1s_4 - c_{23}c_1c_4) + d_2s_1 - d_3s_1 - a_2c_1s_2 - d_4s_{23}c_1;\\
p_y =& d_5(c_1s_4 + c_{23}c_4s_1) - d_6(s_5(c_1c_4 - c_{23}s_1s_4) + s_{23}c_5s_1) - d_2c_1 + d_3c_1 - a_2s_1s_2 - d_4s_{23}s_1;\\
p_z =& d_1 + a_2c_2 + d_6(c_{23}c_5 + s_{23}s_4s_5) + d_4c_{23} + d_5s_{23}c_4.
\end{aligned}$$

c_i, s_i, c_{23}, s_{23} for $i \in \{1, 6\}$, stand for the short notation of $cos(q_i)$, $sin(q_i)$, $cos(q_2 - q_3)$, and $sin(q_2 - q_3)$, respectively.

The geometric Jacobian is $\mathbf{J}_G(\mathbf{q}) = [J_{ij}] \in \mathbf{R}^{6\times 6}$, where

$$
\begin{aligned}
J_{11} =& d_6(s_5(c_1c_4 - c_{23}s_1s_4) + s_{23}c_5s_1) - d_5(c_1s_4 + c_{23}c_4s_1) + d_2c_1 - d_3c_1 + a_2s_1s_2 + d_4s_{23}s_1;\\
J_{12} =& -c_1(a_2c_2 + d_6(c_{23}c_5 + s_{23}s_4s_5) + d_4c_{23} + d_5s_{23}c_4);\\
J_{13} =& c_1(d_6(c_{23}c_5 + s_{23}s_4s_5) + d_4c_{23} + d_5s_{23}c_4);\\
J_{14} =& d_6c_{23}c_1c_4s_5 - d_6s_1s_4s_5 - d_5c_{23}c_1s_4 - d_5c_4s_1;\\
J_{15} =& d_6(s_{23}c_1s_5 + c_4c_5s_1 + c_{23}c_1c_5s_4);\\
J_{16} =& J_{26} = J_{31} = J_{36} = J_{41} = J_{51} = J_{62} = J_{63} = 0;\\
J_{21} =& d_6(s_5(c_4s_1 + c_{23}c_1s_4) - s_{23}c_1c_5) - d_5(s_1s_4 - c_{23}c_1c_4) + d_2s_1 - d_3s_1 - a_2c_1s_2 - d_4s_{23}c_1;\\
J_{22} =& -s_1(a_2c_2 + d_6(c_{23}c_5 + s_{23}s_4s_5) + d_4c_{23} + d_5s_{23}c_4);\\
J_{23} =& s_1(d_6(c_{23}c_5 + s_{23}s_4s_5) + d_4c_{23} + d_5s_{23}c_4);\\
J_{24} =& d_5c_1c_4 + d_6c_1s_4s_5 - d_5c_{23}s_1s_4 + d_6c_{23}c_4s_1s_5;\\
J_{25} =& d_6(s_{23}s_1s_5 - c_1c_4c_5 + c_{23}c_5s_1s_4);\\
J_{32} =& d_4c_2s_3 - a_2s_2 - d_4c_3s_2 + d_5c_2c_3c_4 + d_6c_2c_5s_3 - d_6c_3c_5s_2 + d_5c_4s_2s_3 + d_6c_2c_3s_4s_5 + d_6s_2s_3s_4s_5;\\
J_{33} =& d_4s_{23} - d_5c_{23}c_4 + d_6s_{23}c_5 - d_6c_{23}s_4s_5;\\
J_{34} =& -s_{23}(d_5s_4 - d_6c_4s_5);\\
J_{35} =& d_6s_{23}c_5s_4 - d_6c_{23}s_5;\\
J_{42} =& s_1;\\
J_{43} =& -s_1;\\
J_{44} =& -s_{23}c_1;\\
J_{45} =& c_{23}c_1c_4 - s_1s_4;\\
J_{46} =& s_5(c_4s_1 + c_{23}c_1s_4) - s_{23}c_1c_5;\\
J_{52} =& -c_1;\\
J_{53} =& c_1;\\
J_{54} =& -s_{23}s_1;\\
J_{55} =& c_1s_4 + c_{23}c_4s_1;\\
J_{56} =& -s_5(c_1c_4 - c_{23}s_1s_4) - s_{23}c_5s_1;\\
J_{61} =& 1;\\
J_{64} =& c_{23};\\
J_{65} =& s_{23}c_4;\\
J_{66} =& c_{23}c_5 + s_{23}s_4s_5.
\end{aligned}
$$

The analytical Jacobian is $\mathbf{J}_a(\mathbf{q}) = [J_{a_{ij}}] \in \mathbf{R}^{6\times 6}$, where

$$
\begin{aligned}
J_{a_{11}} =& d_6(s_5(c_1c_4 - c_{23}s_1s_4) + s_{23}c_5s_1) - d_5(c_1s_4 + c_{23}c_4s_1) + d_2c_1 - d_3c_1 + a_2s_1s_2 + d_4s_{23}s_1;\\
J_{a_{12}} =& -c_1(a_2c_2 + d_6(c_{23}c_5 + s_{23}s_4s_5) + d_4c_{23} + d_5s_{23}c_4);\\
J_{a_{13}} =& c_1(d_6(c_{23}c_5 + s_{23}s_4s_5) + d_4c_{23} + d_5s_{23}c_4);\\
J_{a_{14}} =& d_6c_{23}c_1c_4s_5 - d_6s_1s_4s_5 - d_5c_{23}c_1s_4 - d_5c_4s_1;\\
J_{a_{15}} =& d_6(s_{23}c_1s_5 + c_4c_5s_1 + c_{23}c_1c_5s_4);\\
J_{a_{16}} =& J_{a_{26}} = J_{a_{31}} = J_{a_{36}} = J_{a_{41}} = J_{a_{51}} = 0;\\
J_{a_{21}} =& d_6(s_5(c_4s_1 + c_{23}c_1s_4) - s_{23}c_1c_5) - d_5(s_1s_4 - c_{23}c_1c_4) + d_2s_1 - d_3s_1 - a_2c_1s_2 - d_4s_{23}c_1;\\
J_{a_{22}} =& -s_1(a_2c_2 + d_6(c_{23}c_5 + s_{23}s_4s_5) + d_4c_{23} + d_5s_{23}c_4);\\
J_{a_{23}} =& s_1(d_6(c_{23}c_5 + s_{23}s_4s_5) + d_4c_{23} + d_5s_{23}c_4);\\
J_{a_{24}} =& d_5c_1c_4 + d_6c_1s_4s_5 - d_5c_{23}s_1s_4 + d_6c_{23}c_4s_1s_5;\\
J_{a_{25}} =& d_6(s_{23}s_1s_5 - c_1c_4c_5 + c_{23}c_5s_1s_4);\\
J_{a_{32}} =& d_4c_2s_3 - a_2s_2 - d_4c_3s_2 + d_5c_2c_3c_4 + d_6c_2c_5s_3 - d_6c_3c_5s_2 + d_5c_4s_2s_3 + d_6c_2c_3s_4s_5 + d_6s_2s_3s_4s_5;\\
J_{a_{33}} =& d_4s_{23} - d_5c_{23}c_4 + d_6s_{23}c_5 - d_6c_{23}s_4s_5;\\
J_{a_{34}} =& -s_{23}(d_5s_4 - d_6c_4s_5);\\
J_{a_{35}} =& d_6s_{23}c_5s_4 - d_6c_{23}s_5;\\
J_{a_{42}} =& (c_\gamma s_1 - s_\gamma c_1)/(c_\beta(c_\gamma^2 + s_\gamma^2));\\
J_{a_{43}} =& -(c_\gamma s_1 - s_\gamma c_1)/(c_\beta(c_\gamma^2 + s_\gamma^2));\\
J_{a_{44}} =& -(s_{23}(c_\gamma c_1 + s_\gamma s_1))/(c_\beta(c_\gamma^2 + s_\gamma^2));\\
J_{a_{45}} =& (s_\gamma(c_1s_4 + c_{23}c_4s_1))/(c_\beta(c_\gamma^2 + s_\gamma^2)) - (c_\gamma(s_1s_4 - c_{23}c_1c_4))/(c_\beta(c_\gamma^2 + s_\gamma^2));\\
J_{a_{46}} =& (c_\gamma(s_5(c_4s_1 + c_{23}c_1s_4) - s_{23}c_1c_5))/(c_\beta(c_\gamma^2 + s_\gamma^2)) - (s_\gamma(s_5(c_1c_4 - c_{23}s_1s_4) + s_{23}c_5s_1))/(c_\beta(c_\gamma^2 + s_\gamma^2));\\
J_{a_{52}} =& -(c_\gamma c_1 + s_\gamma s_1)/(c_\gamma^2 + s_\gamma^2);\\
J_{a_{53}} =& (c_\gamma c_1 + s_\gamma s_1)/(c_\gamma^2 + s_\gamma^2);\\
J_{a_{54}} =& -(s_{23}(c_\gamma s_1 - s_\gamma c_1))/(c_\gamma^2 + s_\gamma^2);\\
J_{a_{55}} =& (c_\gamma(c_1s_4 + c_{23}c_4s_1))/(c_\gamma^2 + s_\gamma^2) + (s_\gamma(s_1s_4 - c_{23}c_1c_4))/(c_\gamma^2 + s_\gamma^2);\\
J_{a_{56}} =& -(c_\gamma(s_5(c_1c_4 - c_{23}s_1s_4) + s_{23}c_5s_1))/(c_\gamma^2 + s_\gamma^2) - (s_\gamma(s_5(c_4s_1 + c_{23}c_1s_4) - s_{23}c_1c_5))/(c_\gamma^2 + s_\gamma^2);\\
J_{a_{61}} =& 1;\\
J_{a_{62}} =& (s_\beta(c_\gamma s_1 - s_\gamma c_1))/(c_\beta(c_\gamma^2 + s_\gamma^2));\\
J_{a_{63}} =& -(s_\beta(c_\gamma s_1 - s_\gamma c_1))/(c_\beta(c_\gamma^2 + s_\gamma^2));\\
J_{a_{64}} =& c_{23} - (c_\gamma s_\beta s_{23}c_1)/(c_\beta(c_\gamma^2 + s_\gamma^2)) - (s_\beta s_\gamma s_{23}s_1)/(c_\beta(c_\gamma^2 + s_\gamma^2));\\
J_{a_{65}} =& s_{23}c_4 - (c_\gamma s_\beta(s_1s_4 - c_{23}c_1c_4))/(c_\beta(c_\gamma^2 + s_\gamma^2)) + (s_\beta s_\gamma(c_1s_4 + c_{23}c_4s_1))/(c_\beta(c_\gamma^2 + s_\gamma^2));\\
J_{a_{66}} =& c_{23}c_5 + s_{23}s_4s_5 + (c_\gamma s_\beta(s_5(c_4s_1 + c_{23}c_1s_4) - s_{23}c_1c_5))/(c_\beta(c_\gamma^2 + s_\gamma^2))\\
&- (s_\beta s_\gamma(s_5(c_1c_4 - c_{23}s_1s_4) + s_{23}c_5s_1))/(c_\beta(c_\gamma^2 + s_\gamma^2)).
\end{aligned}
$$

c_γ, c_β, s_γ, s_β, represent the short notation of $cos(\gamma)$, $cos(\beta)$, $sin(\gamma)$, and $sin(\beta)$, respectively. The RPY Euler angles are given by

$$
\begin{aligned}
\psi &= atan2\left(\frac{r_{32}}{cos(\beta)}, \frac{r_{33}}{cos(\beta)}\right);\\
\beta &= atan2\left(-r_{31}, \sqrt{r_{32}^2 + r_{33}^2}\right);\\
\gamma &= atan2\left(\frac{r_{21}}{cos(\beta)}, \frac{r_{11}}{cos(\beta)}\right).
\end{aligned}
$$

References

[1] M. W. Spong, S. Hutchinson, and M. Vidyasagar, *Robot modeling and control*. John Wiley & Sons, 2020.

[2] T. M. Inc., *C++ mex functions*, Natick, Massachusetts, United States, 2024. [Online]. Available: `https://www.mathworks.com/help/matlab/matlab_external/c-mex-functions.html`.

[3] I. Kinova, *Gen3 lite robot*, `https://www.kinovarobotics.com/product/`gen3-lite-robots [Accessed: (Dic 10th, 2024)], 2024.